Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Tuesday 25 June 2019

Morning (Time: 1 hour 30 minutes)	Paper Reference **9FM0/4A**

Further Mathematics
Advanced
Paper 4A: Further Pure Mathematics 2

You must have: Mathematical Formulae and Statistical Tables (Green), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B)
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P 6 1 1 8 3 A 0 1 2 8

Answer ALL questions. Write your answers in the spaces provided.

1. A complex number $z = x + iy$ is represented by the point P in an Argand diagram.

Given that

$$|z - 3| = 4|z + 1| \leftarrow \text{is a circle}$$

(a) show that the locus of P has equation

$$15x^2 + 15y^2 + 38x + 7 = 0$$

(2)

(b) Hence find the maximum value of $|z|$

(3)

(a) $|z - 3| = 4|z + 1|$

$|(x-3) + y| = 4|(x+1) + y|$ ← replace z with $x+y$ and group real / im terms

$(x-3)^2 + y^2 = 4^2\left[(x+1)^2 + y^2\right]$ 1 ← square terms to remove negative variants.

$x^2 - 6x + 9 + y^2 = 16\left[x^2 + 2x + 1 + y^2\right]$

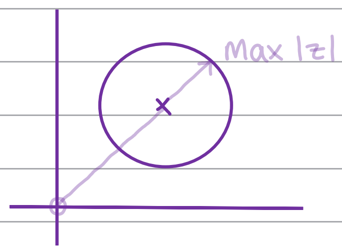
$x^2 - 6x + 9 + y^2 = 16x^2 + 32x + 16 + 16y^2$

$15x^2 + 15y^2 + 38x + 7 = 0$ 1

(b) Complete the square on the x term:

$15\left(x + \frac{19}{15}\right)^2 - \frac{361}{15} + 7 - 15y^2 = 0$ 1

$\therefore$ Centre is $\left(-\frac{19}{15}, 0\right)$



Radius $= \sqrt{\left(\frac{19}{15}\right)^2 - \frac{7}{15}} = \frac{16}{15}$ 1

Max $|z|$ = origin → centre → radius

Max $|z| = \frac{16}{15} + \frac{19}{15} = \frac{7}{3}$ 1

Question 1 continued

(Total for Question 1 is 5 marks)

2. The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(a) Show that 2 is a repeated eigenvalue of **A** and find the other eigenvalue.

(5)

(b) Hence find three non-parallel eigenvectors of **A**.

(4)

(c) Find a matrix **P** such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix.

(2)

(a) Eigenvalues λ_i are such that $Av = \lambda v \Rightarrow (A - \lambda I) = 0$

$\det(A - \lambda I) = 0$

$$\left| \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix} \right| = 0 \quad ①$$

$$0 = (6-\lambda)[(3-\lambda)(3-\lambda) - (-1\times-1)] - (-2)[-2(3-\lambda) - (-1\times 2)]$$
$$+ 2[(-2\times-1) - 2(3-\lambda)] \quad ①$$

$$0 = (6-\lambda)[\lambda^2 - 6\lambda + 9 - 1] + 2[-6 + 2\lambda + 2]$$
$$+ 2[2 - 6 + 2\lambda]$$

$$0 = (6-\lambda)[\lambda^2 - 6\lambda + 8] + 4\lambda - 8 + 4\lambda - 8$$

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Question 2 continued

$0 = 6\lambda^2 - 36\lambda + 48 - \lambda^3 + 6\lambda^2 - 8\lambda + 8\lambda - 16$

$0 = \lambda^3 - 12\lambda^2 + 36\lambda - 32$

Take $\lambda - 2$ as a factor :

$(\lambda - 2)(\lambda^2 - 10\lambda + 16) = 0$ ①

$(\lambda - 2)(\lambda - 2)(\lambda - 8) = 0$

$\therefore\ \lambda - 2 = 0 \Rightarrow \lambda = 2$ is a repeated eigenvalue. ①

$\lambda - 8 = 0 \Rightarrow \lambda = 8$ ①

(b) Use $A\mathbf{v} = \lambda \mathbf{v}$ for each λ :

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 8\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$ ①

$$\left.\begin{aligned} 6x - 2y + 2z &= 8x \\ -2x + 3y - z &= 8y \\ 2x - y + 3z &= 8z \end{aligned}\right\} \quad \begin{aligned} -2x - 2y + 2z &= 0 \quad ① \\ -2x - 5y - z &= 0 \quad ② \\ 2x - y - 5z &= 0 \quad ③ \end{aligned}$$

① − 2③ : $-6x + 12z = 0 \Rightarrow x = 2z$

① + 2② . $-6x - 12y = 0 \Rightarrow x = -2y$

So when $x = 2,\ y = -1,\ z = 1$ $\quad \mathbf{v}_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ ①

Question 2 continued

$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\left.\begin{aligned} 6x - 2y + 2z &= 2x \\ -2x + 3y - z &= 2y \\ 2x - y + 3z &= 2z \end{aligned}\right\} \quad \begin{aligned} 4x - 2y + 2z &= 0 \quad ① \\ -2x + y - z &= 0 \quad ② \\ 2x - y + z &= 0 \quad ③ \end{aligned}$$

All combinations lead to $0 = 0$ so try:

$x = 0$: $-2y + 2z = 0$

$2y = 2z$

$V_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ ①

$y = 0$: $4x + 2z = 0$

$2x = -z$

$V_3 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$ ①

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Question 2 continued

(Total for Question 2 is 11 marks)

3. The number of visits to a website, in any particular month, is modelled as the number of visits received in the previous month plus k times the number of visits received in the month before that, where k is a positive constant.

Given that V_n is the number of visits to the website in month n,

(a) write down a general recurrence relation for V_{n+2} in terms of V_{n+1}, V_n and k.

(1)

For a particular website you are given that
- $k = 0.24$
- In month 1, there were 65 visits to the website.
- In month 2, there were 71 visits to the website.

(b) Show that

$$V_n = 50(1.2)^n - 25(-0.2)^n$$

(5)

This model predicts that the number of visits to this website will exceed one million for the first time in month N.

(c) Find the value of N.

(2)

(a) $V_{n+2} = V_{n+1} + kV_n$ ①

(b) This is a second-order homogenous linear recurrance relation of the form $u_n = au_{n-1} + bu_{n-2}$, so the characteristic equation is $\lambda^2 - a\lambda - b = 0$.

$V_{n+2} = \underset{a}{V_{n+1}} + \underset{b}{kV_n}$

Characteristic Equation. $\lambda^2 - \lambda - k = 0$ $\quad k = 0.24$

$\lambda^2 - \lambda - 0.24 = 0$

$(\lambda - 1.2)(\lambda + 0.2) = 0$

$\lambda = 1.2, -0.2$ ①

Two real roots so general solution has the form $u_n = A\alpha^n + B\beta^n$ where A and B are constants.

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Question 3 continued

General Solution. $V_n = A(1.2)^n + B(-0.2)^n$ 1

In month 1, $V = 65$:

$65 = A(1.2)^1 + B(-0.2)^1$

① $65 = 1.2A - 0.2B$

In month 2, $V = 71$.

$71 = A(1.2)^2 + B(-0.2)^2$

② $71 = 1.44A + 0.04B$ 1

① + 5②. $65 + 5(71) = (1.2 + 5 \times 1.44)A + (-0.2 + 5 \times 0.04)B$

solve simultaneously for A and B

$420 = 8.4A$

$50 = A$

①. $65 = 1.2(50) - 0.2B$

$5 = -0.2B$

$-25 = B$ 1 for A and B

$V_n = 50(1.2)^n - 25(-0.2)^n$ 1

(c) As $n \to \infty$, $-25(-0.2)^n \to 0$ so ignore this term.

$50(1.2)^N > 10^6$ 1 ← 1 million $= 10^6$

$1.2^N = \frac{10^6}{50}$

$N = \log_{1.2}\left(\frac{10^6}{50}\right)$

$N = 54.3$ so exceeds 1 million at the start of month 55. 1

(Total for Question 3 is 8 marks)

4. (i) Use Fermat's Little Theorem to find the least positive residue of 6^{542} modulo 13
(5)

(ii) Seven students, Alan, Brenda, Charles, Devindra, Enid, Felix and Graham, are attending a concert and will sit in a particular row of 7 seats. Find the number of ways they can be seated if

(a) there are no restrictions where they sit in the row,
(1)

(b) Alan, Enid, Felix and Graham sit together,
(2)

(c) Brenda sits at one end of the row and Graham sits at the other end of the row,
(2)

(d) Charles and Devindra do not sit together.
(2)

(i) If p is prime and a is not divisible by p then.

- $a^{p-1} \equiv 1 \bmod p$
- $a^{p} \equiv a \bmod p$

$6^{13-1} \equiv 1 \bmod 13$ ①

$542 - 45 \times (13-1) = 2$ ① ← there are 45 lots of 12 in 542, with 2 remaining

$6^{542} = (6^{12})^{45} \times 6^2$ ①

$1 \times 6^2 = 10 \bmod 13$ ① ← $6^{12} = 1 \bmod 13$, so we need to find the value of $6^2 \bmod 13$.

$\therefore 6^{542} = 10 \bmod 13$ ①

(ii)(a) permutations of n items with no ordering $= n!$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$ ①

(ii)(b) $4! \times 4! = 576$ ②

then there are 3 inividuals + 1 block = 4 units to arrange.

treat the 4 students as a 'block' in which there are 4! ways to order them.

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Question 4 continued

(ii)(c) There are 2! ways B and G can sit

$7 - 2 = 5$ so there are 5! ways everyone else can sit

$2! \times 5! = 2 \times 120 = 240$ 2

(ii)(d) $7! - 6! \times 2! = 3600$ 2

all possibilities.   there are $6! \times 2!$ ways in which they could sit together (same method as (ii)(b)).

(Total for Question 4 is 12 marks)

5.

indicates that you need to use reduction → $I_n = \int \operatorname{cosec}^n x \, dx \qquad n \in \mathbb{Z}$

(a) Prove that, for $n \geqslant 2$

$$I_n = \frac{n-2}{n-1} I_{n-2} - \frac{\operatorname{cosec}^{n-2} x \cot x}{n-1}$$

(4)

(b) Hence show that

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \operatorname{cosec}^6 x \, dx = \frac{56}{135}\sqrt{3}$$

(4)

(a) $I_n = \int \csc^n x \, dx$

$= \int \csc^2 x \times \csc^{n-2} x \, dx$

Integration by parts: $\int uv' = uv - \int vu'$

$u = \csc^{n-2} x$ } chain rule | $v' = \csc^2 x$ ①

$p = q^{n-2}$ $\quad p' = (n-2)q^{n-3}$ | $v = -\cot x$

$q = \csc x$ $\quad q' = -\csc x \cot x$

$u' = (n-2)\csc^{n-3} x(-\csc x \cot x)$

$I_n = -\csc^{n-2} x \cot x - \int (n-2)\csc^{n-3} x(-\csc x \cot x)(-\cot x)\, dx$ ①

$I_n = -\csc^{n-2} x \cot x - (n-2)\int \csc^{n-2} x \cot^2 x \, dx$

$I_n = -\csc^{n-2} x \cot x - (n-2)\int \csc^{n-2} x(\csc^2 x - 1)\, dx$

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Question 5 continued

$$I_n = -\csc^{n-2}x\cot x - (n-2)\int \csc^n x\,dx + (n-2)\int \csc^{n-2}x\,dx$$

$$I_n = -\csc^{n-2}x\cot x - (n-2)I_n + (n-2)I_{n-2} \quad ①$$

$$(n-1)I_n = -\csc^{n-2}x\cot x + (n-2)I_{n-2}$$

$$I_n = \frac{(n-2)}{(n-1)}I_{n-2} - \frac{\csc^{n-2}x\cot x}{(n-1)} \quad ①$$

(b) $$I_6 = \frac{6-2}{6-1}I_4 - \frac{\csc^4 x\cot x}{6-1}$$

$$I_6 = \frac{4}{5}I_4 - \frac{\csc^4 x\cot x}{5} \quad ①$$

$$I_6 = \frac{4}{5}\left[\frac{2}{3}I_2 - \frac{\csc^2 x\cot x}{3}\right] - \frac{\csc^4 x\cot x}{5} \quad ①$$

$$I_6 = \frac{4}{5}\left[\frac{2}{3}\left(-\cot x\right) - \frac{\csc^2 x\cot x}{3}\right] - \frac{\csc^4 x\cot x}{5}$$

$$I_6 = \frac{8}{15}\left[-\cot x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} - \left[\frac{4\csc^2 x\cot x}{15}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} - \left[\frac{\csc^4 x\cot x}{5}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \quad ①$$

$$I_6 = \frac{8}{15}\left(\frac{\sqrt{3}}{3}\right) + \frac{16\sqrt{3}}{135} + \frac{16\sqrt{3}}{135}$$

$$I_6 = \frac{56}{135}\sqrt{3} \quad ①$$

Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

6. (i) A binary operation * is defined on positive real numbers by

$$a * b = a + b + ab$$

Prove that the operation * is associative.

(4)

(ii) The set $G = \{1, 2, 3, 4, 5, 6\}$ forms a group under the operation of multiplication modulo 7

(a) Show that G is cyclic.

(2)

The set $H = \{1, 5, 7, 11, 13, 17\}$ forms a group under the operation of multiplication modulo 18

(b) List all the subgroups of H.

(3)

(c) Describe an isomorphism between G and H.

(3)

(i) Associative means $(a * b) * c = a * (b * c)$

$(a * b) * c = (a + b + ab) * c$

$= \underbrace{a + b + ab}_{\text{'a'}} + \underbrace{c}_{\text{'b'}} + \underbrace{(a + b + ab)}_{\text{'a'}}\underbrace{c}_{\text{'b'}}$ 1

$= a + b + ab + c + ac + bc + abc$

factor a out

$= a + b + c + bc + a(b + c + bc)$ 1

$a * (b * c) = a * (b + c + bc)$

$= a + b + c + bc + a(b + c + bc)$ 1

$(a * b) * c = a * (b * c)$ so * is associative 1

Question 6 continued

(ii)(a) $3^2 = 3 \times 3 \bmod 7 = 9 \bmod 7 = 2$

$3^3 = 2 \times 3 \bmod 7 = 6 \bmod 7 = 6$

$3^4 = 6 \times 3 \bmod 7 = 18 \bmod 7 = 4$

$3^5 = 4 \times 3 \bmod 7 = 12 \bmod 7 = 5$

$3^6 = 5 \times 3 \bmod 7 = 15 \bmod 7 = 1$

3 has order 6 and so generates the group.

$\therefore$ G is cyclic.

(ii)(b) • Groups must contain the identity element (1), all elements must have an inverse

• Subgroups must be closed.

• Lagrange's Theorem states that if H is a subgroup of G, then $|H|$ divides $|G|$

Trivial subgroups are $\{1\}$ and $\{H\}$ 1

$|H| = 6$ so $|h| = 1, 2, 3$ or 6

$\{1, 17\}$ 1 is the only closed subgroup of order 2

$\{1, 17, 13\}$ 1 is the only closed subgroup of order 3

Question 6 continued

(ii)(c)

G	1	2	3	4	5	6
H	1	7	5	13	11	17

identity elements are the same + inverse pairs match up, self-inverse elements match up

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Question 6 continued

(Total for Question 6 is 12 marks)

7. A transformation from the z-plane to the w-plane is given by

$$w = \frac{3iz - 2}{z + i} \qquad z \neq -i$$

(a) Show that the circle C with equation $|z + i| = 1$ in the z-plane is mapped to a circle D in the w-plane, giving a Cartesian equation for D.

(4)

(b) Sketch C and D on Argand diagrams.

(2)

(a) $w = \dfrac{3iz-2}{z+i}$ $\Big\downarrow$ rearrange to get $z = ..$

$zw + iw = 3iz - 2$

$2 + iw = 3iz - zw$

$2 + iw = (3i - w)z$

$\dfrac{2+iw}{3i-w} = z$ ①

$|z + i| = 1 \implies \left|\dfrac{2+iw}{3i-w} + i\right| = 1$ ①

$\left|\dfrac{2+iw-3-iw}{3i-w}\right| = 1$

$\left|\dfrac{-1}{3i-w}\right| = 1$

$\left|\dfrac{1}{w-3i}\right| = 1$ ①

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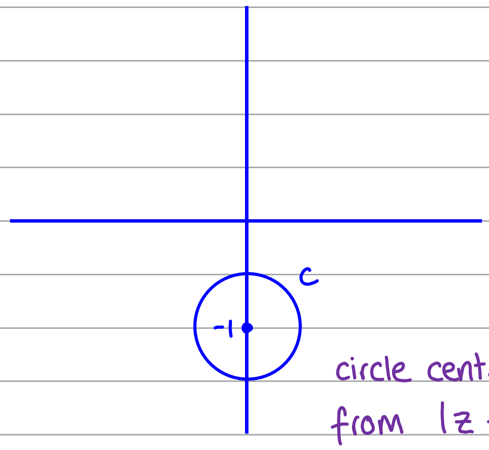
Question 7 continued

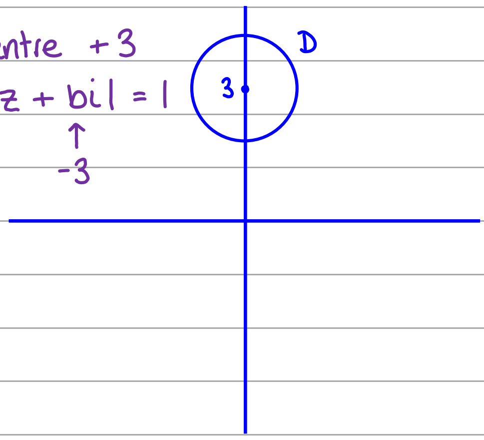
$|w - 3i| = 1$ ← recognise this as a circular locus of points.

$\therefore \; u^2 + (v-3)^2 = 1$ 1

(b)



circle centre +3 from $|z + bi| = 1$ ↑ -3



circle centre -1 from $|z + bi| = 1$ ↑ 1

Question 7 continued

Question 7 continued

(Total for Question 7 is 6 marks)

8.

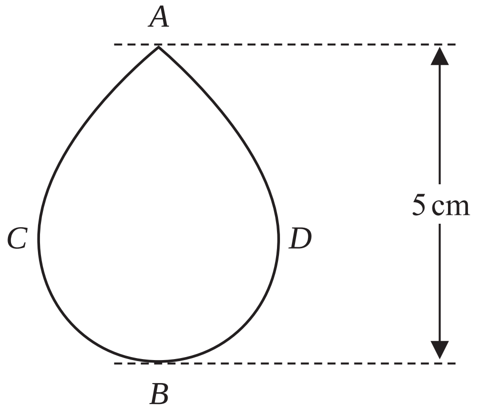


Figure 1

Figure 1 shows the vertical cross section of a child's spinning top. The point A is vertically above the point B and the height of the spinning top is 5 cm.

The line CD is perpendicular to AB such that CD is the maximum width of the spinning top.

The spinning top is modelled as the solid of revolution created when part of the curve with polar equation

$$r^2 = 25\cos 2\theta$$

is rotated through 2π radians about the initial line.

(a) Show that, according to the model, the surface area of the spinning top is

$$k\pi(2 - \sqrt{2})\,\text{cm}^2$$

where k is a constant to be determined.

(7)

(b) Show that, according to the model, the length CD is $\dfrac{5\sqrt{2}}{2}$ cm.

(6)

(a) Surface Area of revolution $S_A = \int_a^b 2\pi r\sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$r^2 = 25\cos 2\theta$ $\quad y = r\sin\theta$, $x = r\cos\theta$

$S_A = \int 2\pi \times r\sin\theta \times \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ 1

$r^2 = 25\cos 2\theta \implies 2r\frac{dr}{d\theta} = -2 \times 25 \times \sin 2\theta = -50\sin 2\theta$

$\frac{dr}{d\theta} = \frac{-50\sin 2\theta}{2r}$ 1

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Question 8 continued

$$S_A = 2\pi \int r\sin\theta \sqrt{25\cos 2\theta + \left(\frac{-50\sin 2\theta}{2r}\right)^2}\, d\theta \quad (1)$$

$$S_A = 2\pi \int r\sin\theta \sqrt{25\cos 2\theta + \frac{2500\sin^2 2\theta}{4r^2}}\, d\theta$$

$$S_A = 2\pi \int 5\sqrt{\cos 2\theta}\,\sin\theta \sqrt{25\cos 2\theta + \frac{2500\sin^2 2\theta}{4(25\cos 2\theta)}}\, d\theta$$

$$S_A = 2\pi \int 5\sqrt{\cos 2\theta}\,\sin\theta \sqrt{25\cos 2\theta + \frac{25\sin^2 2\theta}{\cos 2\theta}}\, d\theta$$

$$S_A = 2\pi \int 5\sqrt{\cos 2\theta}\,\sin\theta \sqrt{25\left(\frac{\cos^2 2\theta}{\cos 2\theta} + \frac{\sin^2 2\theta}{\cos 2\theta}\right)}\, d\theta$$

$$S_A = 2\pi \int 5\sqrt{\cos 2\theta}\,\sin\theta \times 5\sqrt{\frac{\cos^2 2\theta + \sin^2 2\theta}{\cos 2\theta}}\, d\theta$$

$1 = \cos^2\theta + \sin^2\theta$

$$S_A = 2\pi \int 5\sqrt{\cos 2\theta}\,\sin\theta \times \frac{5}{\sqrt{\cos 2\theta}}\, d\theta$$

$$S_A = 2\pi \int 25\sin\theta \, d\theta$$

take out the constant

$$S_A = 50\pi \int \sin\theta \, d\theta \quad (1)$$

Question 8 continued

$S_A = 50\pi \int_0^{\frac{\pi}{4}} \sin\theta \; d\theta$ ①

$S_A = 50\pi \left[-\cos\theta\right]_0^{\frac{\pi}{4}}$ ①

$S_A = 50\pi\left(-\cos\frac{\pi}{4} - -\cos 0\right)$

$S_A = 50\pi\left(-\frac{\sqrt{2}}{2} + 1\right)$

$S_A = 25\pi\left(2 - \sqrt{2}\right)$ ①

(b) $r^2 = 25\cos 2\theta \implies r = 5\sqrt{\cos 2\theta}$

$y = r\sin\theta = 5\sqrt{\cos 2\theta}\,\sin\theta$ ①

Using product rule to find $\frac{dy}{d\theta}$

$u = 5\cos^{\frac{1}{2}} 2\theta \qquad v = \sin\theta$

$u' = \frac{-5\sin 2\theta}{\sqrt{\cos 2\theta}} \qquad v' = \cos\theta$

$uv' + vu' = \frac{-5\sin 2\theta\sin\theta}{\sqrt{\cos 2\theta}} + 5\sqrt{\cos 2\theta}\,\cos\theta$ ①

$\frac{dy}{d\theta} = 0 \implies \frac{-5\sin 2\theta\sin\theta}{\sqrt{\cos 2\theta}} + 5\sqrt{\cos 2\theta}\,\cos\theta = 0$ ①

$\frac{5\sin 2\theta\sin\theta}{\sqrt{\cos 2\theta}} = 5\sqrt{\cos 2\theta}\,\cos\theta$

$\times\sqrt{\cos 2\theta}$

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Question 8 continued

$5\sin 2\theta \sin\theta = 5\cos 2\theta \cos\theta$

use formula booklet to expand double angles

$5(2\cos\theta\sin\theta)\sin\theta = 5(\cos^2\theta - \sin^2\theta)\cos\theta$

$10\sin^2\theta = 5\cos^2\theta - 5\sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$15\sin^2\theta = 5(1-\sin^2\theta)$

$20\sin^2\theta = 5$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ ①

$CD = 2 \times \frac{5}{2} \times \sqrt{\sin\frac{\pi}{6}}$ ① $= \frac{5\sqrt{2}}{2}$ ①

Question 8 continued

(Total for Question 8 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS